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# Application of the Ramberg-Osgood model in asphalt technology

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**Abstract.** The "Time-Temperature Superposition" known from rheology has long been a useful tool for studying the behaviour of asphalt mixture. Dynamic modulus values are measured at different temperatures and frequencies can be thoroughly studied using the master curves defined by using this principle. The master curves are usually constructed using the sigmoid functions. However, other types of functions could be used for this purpose as well. One such option is the Ramberg-Osgood material model designed to model the cyclic behaviour of soils. The present article seeks to find out how accurately the use of the Ramberg-Osgood material model can describe the material behaviour of asphalt mixtures, and if there are any new highlights compared to the commonly used master curve determination techniques.

## 1. Time-Temperature Superposition

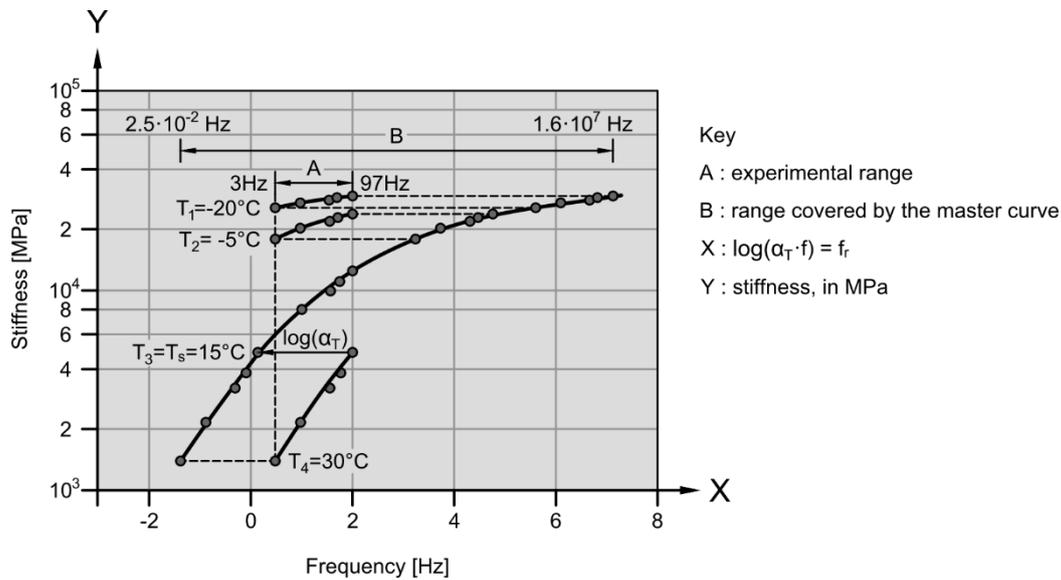
The "time-temperature superposition principle", first formulated by Boltzmann, states that the relaxation time constants of thermorheologically simple materials change to the same degree due to the effect of temperature change. Consequently, the load-time dependent quantities like the complex modulus (which is an important parameter for asphalt mixtures) can be shifted along the time axis in a temperature-dependent rate, therefore the material properties measured at different temperatures can be plotted in a single graph, which called a master curve.

The time-temperature superposition principle allows that the data collected at different temperature values and frequencies to be moved horizontally relative to a reference temperature or frequency, thereby synchronize the various isotherms to form a single master curve.

Annex G of the standard MSZ EN 12697-26: 2018 (Bituminous mixtures. Test methods for hot mix asphalt. Part 26: Stiffness) specifically addresses this issue, stating that: "The stiffness modulus can be determined on the basis of a master curve for the desired temperature at the desired loading time." The standard outlines the principle of the master curve's calculation, and gives an example, which emphasizes that many methods can be used to determine it.

Mathematically, the superposition can be achieved by introducing the so-called reduced frequency ( $f_r$ ). The shift factor  $a_T$  determines the desired shift along the horizontal axis at a given temperature. The actual frequency must be divided by this shift factor to obtain the value of the reduced frequency ( $f_r$ ) in the master curve.





**Figure 1.** Derivation of the master curve.

$$f_r = a_T \cdot f \quad (1)$$

The shift factor at the reference temperature is basically:  $a_T = 1$ .

Studies of the rheological properties of asphalt mixtures in Hungary began in the 1980s (Török, 2000; Gömze, 2005;) and are still ongoing (Gömze et al., 2008; 2009), but it is regrettable that although there are remarkable Hungarian research results, but practical application analysis of master curve that based on asphalt mixtures was not widespread. The article reiterates the importance of the analytical capabilities of the master curves and the applicability of another asphalt model.

## 2. The classic determination method of shift factors

The standard MSZ EN 12697-26 generally describes the principle of defining the master curve. Accordingly, the master curve must be determined at a given temperature by shifting the isotherms plotted at other temperatures (strictly parallel to the load-time / frequency axis). The standard in the example uses the so-called Arrhenius equation to determine the value of the shift factor, and emphasizing that other methods can be used such as the Christensen-Anderson model.

In addition to the Arrhenius equation in rheology, the Williams-Landel-Ferry (WLF) equation is another, most recently used method for determining shift factors. Although both classical shift factors have been used in research of bitumen and asphalt mixtures, they have typically been developed for polymers, but today their application is fading into the background.

However, in recent years, as a result of advances in asphalt technology, several models that developed for asphalt mixtures have created. For example, the value of the shift factor resulting from a large American research program (National Cooperative Highway Research Program: Mechanistic-Empirical Design of New and Rehabilitated Pavement Structures, NCHRP 1-37-A) depends on the temperature-dependent viscosity values of the binder. Nevertheless, many other methods have also been developed.

If large number of measurement results are available, it is possible to treat the shift factor as an independent variable in determining the parameters of the best-fitting relation to the data (using some iteration technique), instead of the classical shift factors. When performing the optimization, the international literature recommends generally the shift factor to define as a quadratic function:

$$\log(a_T) = a \cdot T^2 + b \cdot T + c \quad (2)$$

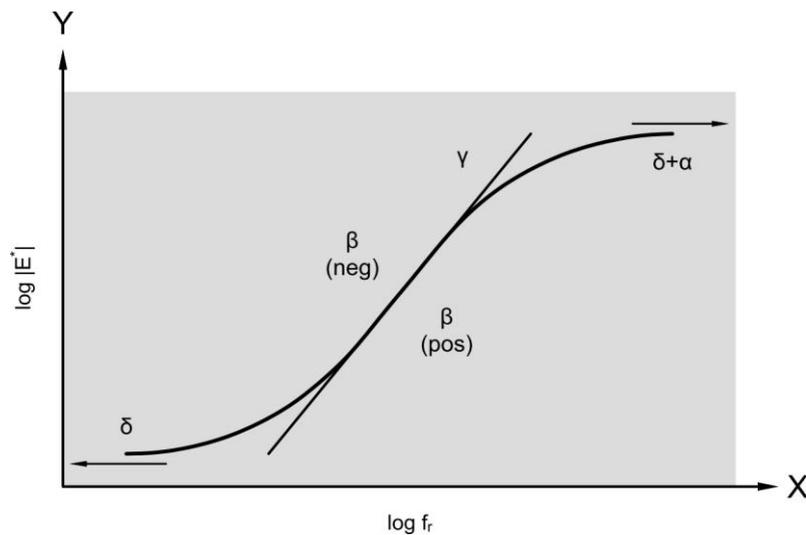
where

$a_T$  : shift factor

$T$  : experimental temperature (K)

$T_{ref}$  : reference temperature (K)

$a, b, c$  : constant (K)



**Figure 2.** Parameterizations of the Sigmoid function.

### 3. Determination of master curve with Sigmoid model

Considering the American researches (see NCHRP 1-37-A, 2004) for the mixture of asphalt, the master curve is a continuous, non-decreasing, and necessarily has lower and upper limits therefore it can be described by a non-linear "s" shaped curve called sigmoid function. Otherwise, the applicability of the function can be verified by physical observation of the behavior of the mixture. Its general formula is as follows:

$$f(x) = \frac{1}{1+e^{(-ax)}} \quad (3)$$

Thus, once the measurement results are available, the master curve of the mixture can be plotted, which can be described with an equation (eq. 4) as a function of frequency in the case of asphalt mixtures based on the research results mentioned above:

$$|E^*| = \delta + \frac{\alpha}{1+e^{\beta-\gamma(\log(fr))}} \quad (4)$$

where

$|E^*|$  : stiffness (MPa)

$\delta, \gamma, \alpha, \beta$  : constant parameters

$t_r$  : reduced load time (s)

$f_r$  : reduced frequency (Hz)

In eq. 4, " $\delta$ " is the minimum value of " $|E^*|$ ", " $\delta + \alpha$ " is the maximum value of " $|E^*|$ ", and " $\beta$ " and " $\gamma$ " are parameters of the sigmoidal function. Figure 2 shows a graphical interpretation of the mentioned parameters. A major disadvantage of the model is that instead of using the complex modulus ( $E^*$ ) of the mixture, it only uses its absolute value (stiffness), therefore the viscoelastic effect, which can be observed with the help of phase angle, cannot be studied in the mixture behavior.

#### 4. The Ramberg-Osgood (RAMBO) model

Master curves are primarily created using sigmoid functions, although other types of functions could be used for this purpose. One such possibility is the Ramberg-Osgood material model designed to model the cyclic behavior of soils (Szilvagy, 2018). The general mathematical form of the Ramberg-Osgood function is:

$$x = y + C \cdot y^R \quad (5)$$

Kweon (2008) suggests eq 5. in the following version to describe the master curve of asphalt mixtures:

$$f_r = E_n \cdot f_r + C \cdot (E_n \cdot f_r)^R \quad (6)$$

where

$f_r$  : reduced frequency,

$E_n$ : normalized dynamic modulus,  $E_n = (E - E_{min}) / (E_{max} - E_{min})$ ,

$C, R$  : parameters.

Like the sigmoid functions, the Ramberg-Osgood model can be fitted to the measurement results using optimization procedures. The result of the fitting is the parameters  $E_{min}$ ,  $E_{max}$ ,  $C$  and  $R$ .

According to Kweon (2008) research, the coefficients of the RAMBO model have an independent effect on the master curve (see Table 1).  $R$  influences the curvature (slope) of the master curve, and  $C$  moves it along the horizontal axis, like the temperature-time shift factor. The latter finding is not examined in detail by the author. The fit of both the sigmoid function and the Ramberg-Osgood (RAMBO) model is affected by the  $a_T$  shift factor used in the calculation of the reduced frequency ( $f_r = a_T \cdot f$ ). As discussed in section 2, several methods have been developed to calculate the  $a_T$  shift factor, however, whichever method is used, the shift factor actually shifts the master curve only parallel to the load duration axis. This effect can also be achieved for the RAMBO model by setting the appropriate value of parameter  $C$ . The magnitude of the shift is a function of the temperature, thus the original RAMBO model can be converted to the following format:

$$f = E_n \cdot f + C_T \cdot (E_n \cdot f)^R \quad (7)$$

where

$f$  : frequency,

$T$  : temperature,

$C_T$  : time factor,

$R$  : slope of the master curve.

The advantage of the proposed model is that the results obtained at different temperatures are used simultaneously to estimate the stiffness of the asphalt mixture with the help of the so-called  $C_T$  time factor.

The use of the time factor is based on the application of the temperature-time superposition principle and can be described by preliminary calculations using the following equation:

$$C_T = a \cdot e^{b \cdot T} \quad (8)$$

where

$a, b$  : regression parameter,

$T$  : temperature.

**Table 1.** The effect of the coefficients on the master curve

Coefficient		Master curve			
		Absolute value		Shape	
		Min.	Max.	Min.	Max.
Sigmoid-function	$a$	●	●	×	×
	$b$	×	●	×	●
	$g$	×	×	●	
	$f$	×	×	×	●
Ramberg-Osgood model	$E_{min}$	●	×	×	×
	$E_{max}$	×	●	×	×
	$R$	×	×	●	×
	$C$	×	×	×	●

●: influencing; ×: not influencing.

### 5. Mixture studies and modeling

In the Department of Highway and Railway Engineering at BME, we investigated the differences between rubber-modified and polymer-modified binder mixtures with each other and with conventional reference binder mixtures. In this study, we compared three asphalt mixtures with the same stone structure:

- AC22 binder 50/70, as reference mixture.
- AC22 binder PmB 25/55-65, as reference mixture.
- AC22 binder GmB 45/80-55, as main investigation mixture.

At this stage of the research, we sought to find out what additional information can be obtained by using master curves based on the stiffnesses of these mixtures.

The complex modulus and phase angle of the three mixtures were determined using SPT (Simple Performance Tester). During the test, stiffness values were set for each of the three mixtures at three temperatures and six different frequencies as follows:

- Temperatures 10°C, 20°C, 30°C.
- Frequencies 0.1 Hz, 0.5 Hz, 1 Hz, 5 Hz, 10 Hz, 25 Hz..

The available data of the complex modulus were processed and published using the Huet-Sayegh model (HS) (Pronk, 2005) (Kisgyörgy et al., 2016).

Based on the results of the HS modelling, it could already be stated that the internal friction of the stone structure correlates to parameter  $E_0$  directly, while the tendency of plastic deformation correlates indirectly, and this  $E_0$  is not only significantly better than conventional bitumen, but exceeds its polymer modified mixtures as well. This is likely to result in favorable rutting resistance at high temperatures. The GmBEinf parameter, which is related to the cold behavior, differs from the other two binder mixtures in a favorable direction, which indicates a slightly cold, i.e., better crack resistance.

As a continuation of the research, we used the previous laboratory results to determine whether the RAMBO model is suitable for studying asphalt mixtures, similar to modelling soil behavior. The data obtained were compared with the results of the sigmoid model.

## 6. Results

The parameters of the RAMBO model can be determined by minimizing the sum of square error (SSE) of the experimental and model estimated frequencies:

$$SSE = \sum \frac{(f_{exp} - f_{pre})^2}{f_{exp}^2} \quad (9)$$

where

$f_{exp}$ : frequency set by the experiment,

$f_{pre}$ : frequency estimated by RAMBO model.

$$f_{pre} = \left[ \frac{C_T \cdot E_n^R}{1 - E_n} \right]^{\frac{1}{1-R}} \quad (10)$$

The optimization method results in the following model parameters  $E_{min}$ ,  $E_{max}$ ,  $a$ ,  $b$  and  $R$ . The goodness-of-fit ( $R^2$ ) of the RAMBO model can be calculated using both estimated frequencies and modules. In practice, the latter is widespread, although frequencies are much more sensitive to model errors. The results of the 6 asphalt concrete mixture are summarized in Table 2.

**Table 2.** Modulus of RAMBO model for 3 types of asphalt mixtures

Mixture type	Code	$E_{min}$	$E_{max}$	$R$	$a$	$b$	$R^2$ -Freq.	$R^2$ -Mod.
AC22 binder 50/70	AG1	1313.57	40715.47	0.6584	0.1331	0.1142	0.6471	0.9819
	AG2	1515.01	32673.32	0.6351	0.1304	0.0985	0.7562	0.9878
AC22 binder PmB 25/55-65	BG1	2072.30	29484.26	0.6266	0.0902	0.1124	0.9943	0.9974
	BG2	1750.00	34667.42	0.6661	0.1538	0.0997	0.8052	0.9945
AC22 binder GmB 45/80-55	CG1	1686.62	25336.58	0.6351	0.3069	0.0958	0.9734	0.9976
	CG2	1638.57	29981.01	0.6669	0.4304	0.0894	0.9748	0.9969

The accuracy of the model is well illustrated by  $R^2$  calculated from estimated and set frequencies. Where  $R^2$  is less than 0.9, there are outliers in the data set, which is probably measurement error.

One of the basic assumptions of the traditional master curve definition is to "fix" the stiffness measured at a selected temperature (reference) by setting the shift factor to value 1 and then move the stiffness measured at different temperatures. We did not make this constraint when using the RAMBO model, thus we obtained the best fit for different mixtures at different temperatures during model fitting. Applying that the  $C_T$  time factor at the "reference temperature" of the asphalt-concrete mixture is equal to one:

$$C_T = 1 \quad (11)$$

Thus, a new reference temperature can be calculated according to the following equation:

$$T_{ref} = -\ln(a) \cdot b^{-1} \quad (12)$$

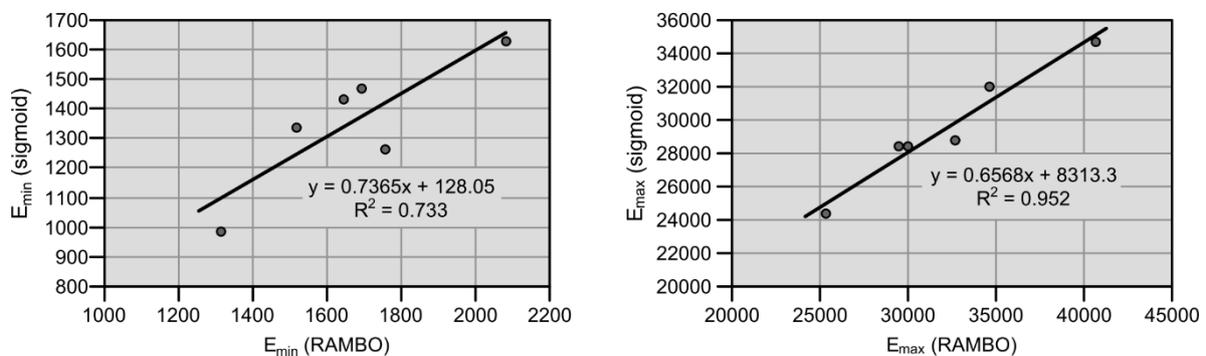
The  $T_{ref}$  temperatures are also listed in Table 2 for each mixture. Its significance and theoretical interpretation require further research, but the first results suggest significant material-specific behavior. For rubber modified mixtures the  $T_{ref}$  is 9-12°C whereas for conventional and polymer modified mixtures it is 19-22°C. A lower  $T_{ref}$  value assumes better cold behavior.

The master curves obtained with the RAMBO model were also compared with those obtained with the sigmoid model. Following the fitting of the sigmoid model using the Arrhenius's shift factor and treating its C parameter as a variable during optimization, the results have been found on Table 3. The last column of the table shows the correlation between the measured and estimated stiffness. Graphically comparing the results of the RAMBO model with the conventional sigmoid model, Figure

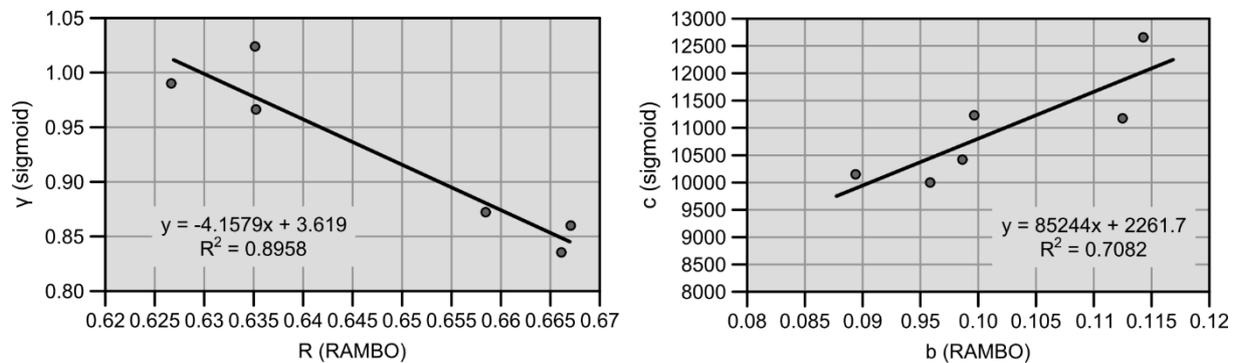
3 shows the theoretical minimum and maximum stiffness, respectively. However, the degree of correlation is different, although the existence of a relationship is confirmed.

**Table 3.** Modulus of Sigmoid model for 3 types of asphalt mixtures

Code	$E_{min}$	$E_{max}$	$C$	$\gamma$	$\beta$	$\alpha$	$\delta$	$R^2$
AG1	987.33	34664.01	12635.20	0.87	-1.14	1.55	2.99	0.9856
AG2	1337.03	28774.68	10416.65	1.02	-1.35	1.33	3.13	0.9909
BG1	1628.32	28363.27	11162.33	0.99	-1.20	1.24	3.21	0.9978
BG2	1263.87	32029.34	11221.79	0.84	-1.12	1.40	3.10	0.9956
CG1	1467.52	24353.73	9985.87	0.97	-0.13	1.22	3.17	0.9983
CG2	1431.53	28363.06	10142.41	0.86	-0.03	1.30	3.16	0.9977



**Figure 3.** Relationship between the Sigmoid and the RAMBO mode, part I.



**Figure 4.** Relationship between the Sigmoid and the RAMBO mode, part II.

Figure 4 shows the relationship between similar parameters. There is also a close relationship between the  $\gamma$  and  $R$  values associated with the slope of the master curve. Significant correlation can also be demonstrated between the material specific parameter ( $C$ ) of the Arrhenius shift factor and one of the RAMBO shift factor ( $b$ ) parameters. It can be seen that the master curves obtained with the help of the two models are very similar.

## 7. Summary

As mentioned above several times, with a single stiffness value at the specified temperature, mixtures can be characterized, but differences between mixtures cannot be fully explored. This is certainly not a new idea. A prominent working group (CROW-report, 2006), in its recommendations to the new Dutch asphalt pavement design system emphasized that the master curve is recommended to be defined, because of its valuable information content.

In this article a possible method for determining the asphalt mixture master curve has been introduced, besides the sigmoid function, called RAMBO model. Temperature-time equivalence offers the potential to study time and frequency ranges that cannot be or difficult to implement experimentally, therefore the development and refinement of the mathematical tools required for this is an important task. With their help, the master curve of asphalt mixtures enables comparisons to be made between tests at different frequencies and temperatures, and furthermore the physical behavior of the asphalt mixture that observed during stiffness tests on the full temperature scale can also be determined.

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