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László Varga – Arianna Kitzinger – Réka Kissné Zsámboki

Changing Perspectives and Attitudes on Early Childhood Research and Education

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Réka KISSNÉ ZSÁMBOKI

TEACHING MATHS AT EARLY YEARS – FROM LECTURE METHOD TO DISCOVERY APPROACH

1. Activity based learning at early years

Early years play a crucial role in the complex and harmonious development of personality, in early cognitive processes and in social learning. Children's individual development is determined by genetic conditions, the specific features of maturation and the spontaneous and planned environmental effects. A young child is a self-constructing personality, whose development requires favourable conditions in which movement, play, activity, curiosity and children's interest can play an essential role. In this process, the child will also gain rich mathematical experience and develop numerical skills and competences. The purpose of this work is to draw attention to the importance and necessity of activity based learning and teaching in the light of the theoretical background, early mathematical skills and teaching methods of pre-primary education.

Learning objectives

There is rich national and international literature on the scientific approach to child development. The concept of biological maturation includes that development has a genetically determined characteristic of human nature. Contrary to the theory that emphasizes maturation dominance, the theory of learning states that development in a childhood personality and activity is appropriate in a good environment (Pléh, 2010). They believe that the individual is shaped by his or her environment throughout his or her life, through reward and punishment. Learning theory reflects a high degree of optimism in terms of teachability, but also conveys the hidden message that if the child's development is inadequate, then his or her environment may be responsible (Pléh, 2010). The 21st century learning theories explain the need to provide the child with a stimulus-rich material environment suitable for exploration. We need to allow children to act independently, which helps them learn.

1.1. Theories about learning

According to the creator of social learning theory (Albert Bandura, Canadian psychologist), observing the behaviour, emotions, and attitudes of others is an important form of children's learning. The active attention of the child during social learning is essential. It has been observed that children only imitate positive models that are important to them. Therefore, we must constantly strive to improve our behaviour, thinking, relationships, etc. to give good patterns for children to imitate (Pléh, 2010).

Perhaps the most interesting theories for the development and development of mathematical competences are the concepts of neuropedagogy, which emphasizes cognitive change and the new science of cognitive neuroscience. Jean Piaget, a Swiss psychologist, developed his theory that thinking is the cause of development through observation of his own children and many developmental psychological experiments. At the heart of the theory is the child's worldview, a scheme that the child constructively builds with constant thinking. When they encounter a new phenomenon, they insert it into their previous knowledge (assimilation) or reshapes their worldview so that the new phenomenon can now be explained (accommodation). This continuous adaptation, that is, learning takes place throughout our lives.

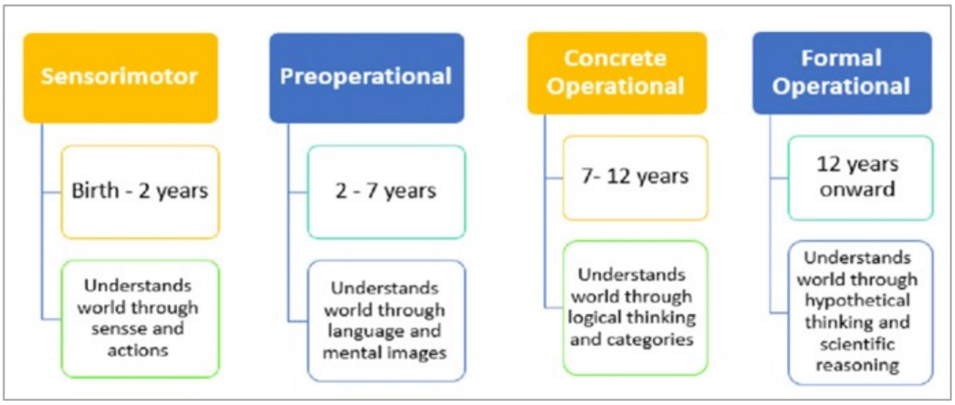


Figure 1. Jean Piaget's learning theory
(https://bit.ly/3N81bu2)

Neurology, cognitive neuroscience, and neuropedagogy teach that the basic structure of the brain is built through a long-term developmental process that begins in fetal life. Brain plasticity is highest in early childhood (the first 5-6 years). More specialized brains later find it increasingly difficult to adapt to very new or unexpected challenges (Schiller, 2010; Varga, 2015). Research from recent decades shows that we are all born with a "programmed" brain that is fundamentally personal and already possesses some basic information about the human and material world in infancy (Klingberg, 2012). Although a child's brain mass is four times smaller than that of an adult, a newborn child's mind contains almost all the neurons that they will use later in life. Growth is facilitated by the formation of a complex network of intercellular protrusions that require a great deal of individual experience. According to Donald Hebb's "Fire Together - Wire Together" theory, the effect of stimuli causes cells with similar functions to signal and begin to project to those that radiate their own signals at the same time. Based on the above, the network of neurons is not only random and not pre-programmed but is formed by experience. Of the synapses that occur, only those that are used regularly remain permanently, and the rest are lost through synaptic backlash (Keysers & Gazzola, 2014).

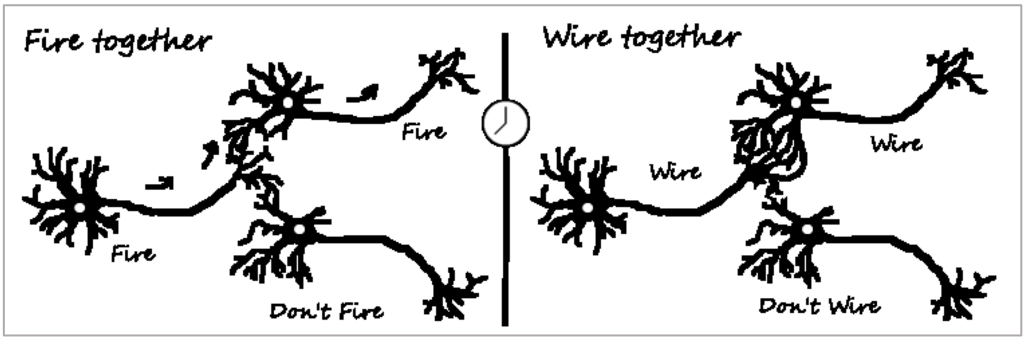


Figure 2. Donald Hebb's "Fire Together - Wire Together" theory
(https://bit.ly/3uiLuaL)

In addition to this basic knowledge, we are born with a very effective learning ability and a basic need for learning. In a sociocultural defined learning process, the children rely not only on their own observations and experiences, but also on the interaction with those around them (Kitzinger, 2009). Thus, the quality of human relationships is crucial for children's learning and development. Daniel N. Stern, a prominent figure in twentieth-century attachment theory, considers social relationships to be decisive in children's learning. Factors and components that determine a child's personality, such as the development of a positive self-image, a sense of confidence and security, and even self-reflection, are highly dependent on the positive or negative relationships and experiences that are experienced. Positive, credible, direct relationships with children have the power to promote their development in all areas. Thus, the experience gained in early childhood, the characteristics of family life, and the quality of environmental (pedagogical) stimulation clearly influence the healthy development of children and its cognitive, emotional, and social well-being (Stern, 2002).

Comprehensive questions:

- What does social learning mean by A., Bandura?
- How did neuropedagogy indicate paradigmatic changes of scientific theories on children's brain development?
- How could you summarise Donald Hebb's "Fire Together - Wire Together" theory?
- What are the main factors and components determining a child's personality?

2. Early math skills

"Early math skills have the greatest predictive power, followed by reading and then attention skills" – reports a psychology squad led by Greg J. Duncan, in School readiness and later achievement.[8] Follow-up studies continue to confirm the importance of early math skills. *"Math is the language of logic* […] *builds reasoning, which leads to comprehension.* […] *Developing a mentally organized way of thinking is critical"* - as it is stated in a research published by Jie-Qi Chen, professor of Child Development at the Erikson Institute.[9] *"Mathematics is the creation of the human mind, at the same time the product of the evolution of the brain and culture,"*[10] says Valéria Csépe, research professor at the Hungarian Academy's Brain Imaging Center. For centuries, the science of early childhood development and mathematics were very distant and incompatible concepts. Thanks to the emergence of the field of intelligence and mental processes, and the emergence of cognitive psychology, we can now speak of mathematical discovery and experience from birth[11].

8 published in Developmental Psychology in 2007
9 URL: https://www.greatschools.org/gk/articles/early-math-equals-future-success [2022.03.22.]
10 URL: http://mipszi.hu/cikk/091212-szamolo-agy [2021.02.01.]
11 scientific theories of Jean PIAGET, Tamás VARGA and György PÓLYA

Early Math Key Concept	Impact on Future
Number Sense	*Counting backwards *Adding and Subtracting
Patterns	*Make predictions *Reasoning skills *Logical connections
Representation	*Make mathematical concepts real through objects, pictures, symbols, and words
Measurement	*Find length, height, width, and weight *Tell time
Spatial Sense	*Shape, size, position, direction, movement *Geometry skills
Estimation	*Makes estimations easier to learn
Problem-Solving	*Use past knowledge and logical thinking to solve new problems

Figure 3. How Math Skills Impact Student Development
(https://study.com/academy/lesson/how-math-skills-impact-student-development.html)

Tamás Varga, a mathematics teacher, an internationally recognized and distinguished expert in mathematics, pointed out in his articles on "Baby Maths" published four decades ago that the baby's connection with its living and non-living environment through its senses, movements and sounds has elements can be considered to be mathematical (Dienes, 2014). He believes that a young child's self-constructing personality - neither movement nor speech development should be rushed, pushed, directed - merely creates favourable conditions for natural development in which movement, play, activity, curiosity interest in children can play a vital role. It is also possible to gain rich mathematical experience in this development. The visual and linguistic information related to the actions can have important mathematical elements and contents (Kissné, 2017). However, to help the development of young children consciously, most often it does not require complex tasks and abstract concepts. From birth, children are ready to learn, naturally curious and motivated to explore the world around them. It is up to the people around them and the stimulating environment to preserve this natural curiosity in the future. Among the child's various activities, it is almost impossible to find one without any direct or indirect mathematical experience. They are unobtrusively acquired by children through experience as they explore the outside world. Thus, for them, the process of learning about mathematics is an interesting and exciting discovery, which in case of success is a good inspiration for further experimentation.

Comprehensive questions:

- How could you justify the importance of early math skills?
- What was the main points of Tamás Varga's conception of baby maths?
- What are the key factors of the stimulating environment for young children?

3. From cognition to conceptualization at early years

3.1. Thinking and learning by doing

Due to the active lifestyle of the toddler in the stimulating environment, the cognitive processes, the observation and recognition reaches the level that by the end of the toddler's life he/she will be able to solve the problematic situations independently. The child develops elements of learning that accompany him or her throughout his or her life. This is important from the point of view of mathematical education, because mathematical abilities are among the earliest manifesting abilities, and in order to develop they require all cognitive processes, perception – attention – memory – imagination – thinking. Therefore, mathematical education must pay attention to the development and improvement of these cognitive processes.

Developing such thinking in mathematics in early childhood is one of the most important goals of mathematics education at almost all ages. Mathematical knowledge would be self-contained and formal if not linked to by means of thinking. For this, however, the existence of a stimulating, inspirational and exploratory personal and material environment is indispensable. In addition, mapping early skills can be of paramount importance for the success of subsequent learning processes.

According to Stella Lourenco, a senior psychologist at the Children's Study Center at Emory University in Atlanta, United States, when you observe infants' spatial thinking and orientation, there is much to learn about their future/emerging mathematical abilities. The signs of spatial thinking appear at the age of six months, which is clearly related to later mathematical intelligence. It has been found that the ability of spatial reasoning is strongly related to later mathematical performance (Kissné Zsámboki – Farnady Landerl, 2018).

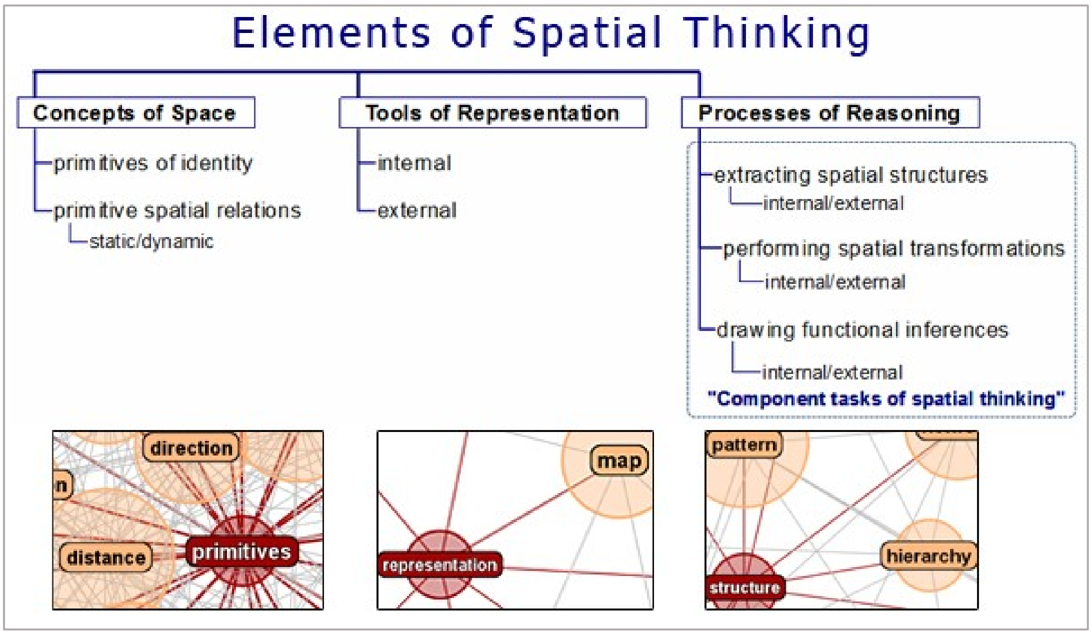


Figure 4. Elements of spatial thinking
(http://teachspatial.org/elements-of-spatial-thinking)

Experience is the result of perception, memory, imagination, without affecting thinking. The process of gaining experience begins with perception. Even a one-year-old child can recognize repeatedly observed things. Recognition is the first form of remembering. In more complex situations it is harder to recognize, and in experiential situations it is easier to remember. Our memories are not always realistic reproductions of perception, but certain qualities are more pronounced in them, and a typical image may be produced by generalization, which reflects reality more deeply than direct perception. When memory/recollection breaks away from reality, we are talking about imaginary imagery, evocation of imagination. This is also the way to gain mathematical experience (Cole & Cole, 2006).

The formation of the thinking structure begins with action. Thus, we can talk about thinking operations from an early age. The highest level of cognitive activity is thinking. It is necessary to solve problems that cannot be solved directly by perception, memory or imagination. The problem is that there is a goal we want to achieve, but we do not know how to reach it. Through these cognitive functions, the toddler develops elements of learning that accompany him or her throughout his or her life. This is important for mathematical education because mathematical abilities are among the earliest manifestations and require all cognitive processes, perception - perception - attention - memory - imagination and thinking to develop. That is why mathematical education must pay attention to the development and development of these cognitive processes (Zsámboki, 2007).

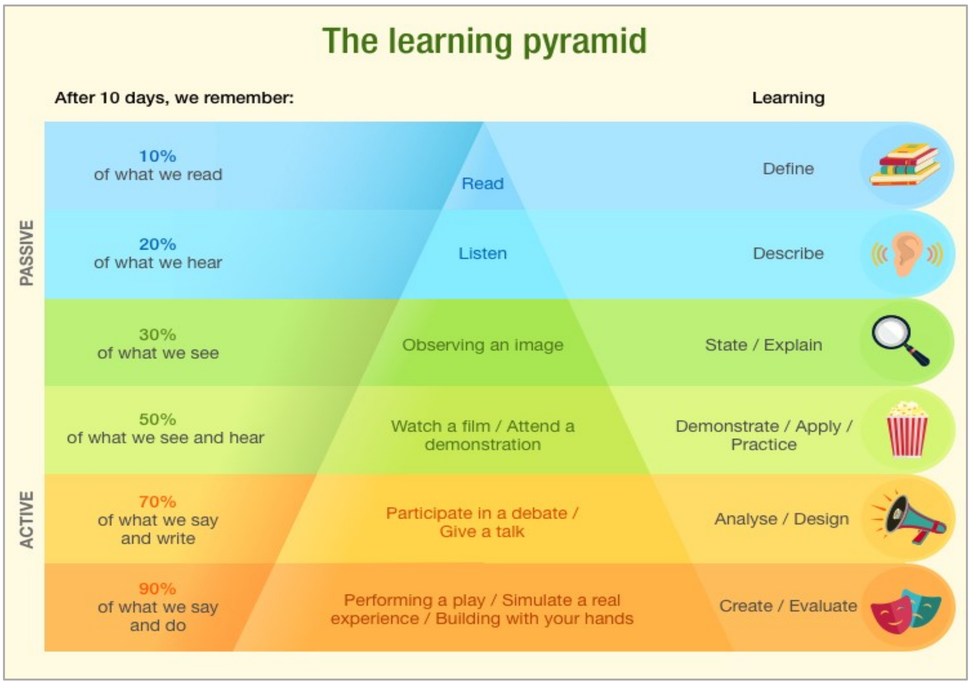


Figure 5. The learning pyramid
(https://www.iberdrola.com/talent/learning-by-doing)

3.2. Developmental stages of conceptualization

Reviewing the development of thinking and conceptualization processes as a result of gaining experience can also be important because, in addition to developing a harmonious personality, gaining experience, educational work can greatly contribute to the development of problem-solving thinking and concepts in children. (For the foundation of both mathematical and non-mathematical concepts.) The description of the developmental stages of conceptualization is related to the name of György Pólya, a world-famous Hungarian mathematician.

Figure 6. Pólya György,
Hungarian mathematician, the father of problem solving[12]
(https://peoplepill.com/people/george-polya)

At the beginning of the developmental stages, gaining experience, gathering facts and information while manipulating objects is of paramount importance. At this stage, it is important that the essential features of a given concept are repeated and the non-essential ones change. Children should be able to participate in many sensory-movement experiences and manipulation with objects and toys appropriate to their age. During the second stage, the experiences, the memories, "come together". Children now observe the typical features of the concept and are able to recognize shapes similar to the concept on the basis of their perceptions. In the third phase, during formalization, imagery becomes knowledge during thought operations (primarily abstraction and generalization). Knowledge is verbalised. Abandoned from all their other qualities, the shapes previously called similar now get the same generic name. This phase can be done later, at the end of kindergarten or early in school. In the final, assimilation phase of conceptualization, concepts are integrated into a coherent system that expands and possibly undergoes structural changes. The concept itself changes as the child becomes aware of his or her place in the given conceptual system and its relation to other systemic elements (Butterworth, 2005).

Comprehensive questions:

- What kind of visual indicators can be detected of spatial thinking appearing at the age of six months?
- How could you find correlation between the 'learning pyramid' model and the concept on learning math at early years?

[12] Pólya György (1887, Budapest – 1985, Palo Alto, California, United States). He was a professor of mathematics from 1914 to 1940 at ETH Zürich in Switzerland and from 1940 to 1953 at Stanford University. He remained Stanford Professor Emeritus for the rest of his life and career. He worked on a range of mathematical topics, including series, number theory, mathematical analysis, geometry, algebra, combinatorics, and probability. He was an Invited Speaker of the ICM in 1928 at Bologna, in 1936 at Oslo, and in 1950 at Cambridge, Massachusetts.

4. Numeracy and numerical innate abilities

According to a study by David C. Geary, a renowned American cognitive development and evolutionary psychologist, in 1995, we have at least: determining the morbidity of a small population (3-4 items), comparing sets without quantities, element counting ability and addition and subtraction up to 3 (Geary, 2001). Counting as a serial ability is thus inherited. One sign of this is that children count before the age of two, even if they are not in the right order. Around the age of three, the acceleration of arithmetic abilities is observed, because children already understand that the names of each number correspond to a certain number. On the other hand, they are capable of distinguishing part whole. Several studies have shown that children already have the concept of addition and subtraction before the age of five (Desoete et al., 2009).

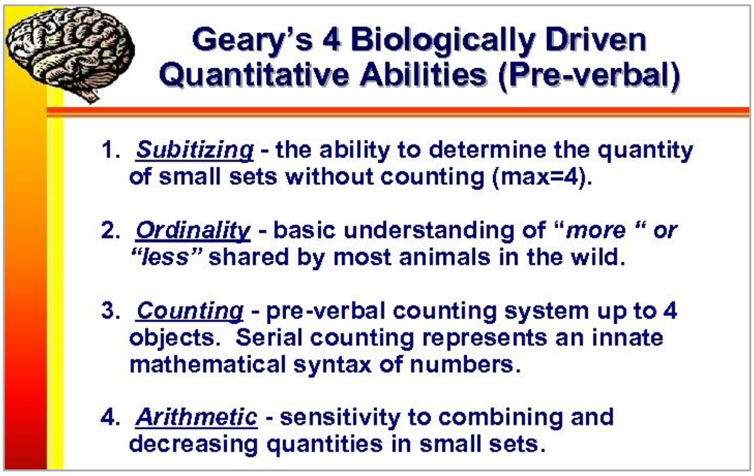


Figure 7. Innate numeric abilities
(https://present5.com/the-neuropsychology-of-mathematics-steven-g-feifer)

Former research points to early addition and subtraction being an innate ability.[13] Subitizing is one of our primary mathematical abilities. "Subitize", from the Latin word for "suddenly", is the ability to quickly identify the number of items in a small group.[14] Toddlers can differentiate between one and three items; by age of seven, this increases to between four and seven items. (Pellissier, 2015)

[13] In a 1992 study at the University of Arizona, for example, 6-month-old babies were shown one baby doll. As the babies watched, a screen was placed in front of the doll and then a second doll was placed behind the screen. When the screen was removed, scientists could tell that, at just 6 months old, babies expected to see two dolls. In instances when there were fewer or more dolls when the screen was removed, the babies stared longer because the results were wrong, a "violation of expectation".

[14] When Dustin Hoffman's character in Rainman looked down at the pile of spilled toothpicks and knew without counting that there were 246, that was an example of advanced subitization.

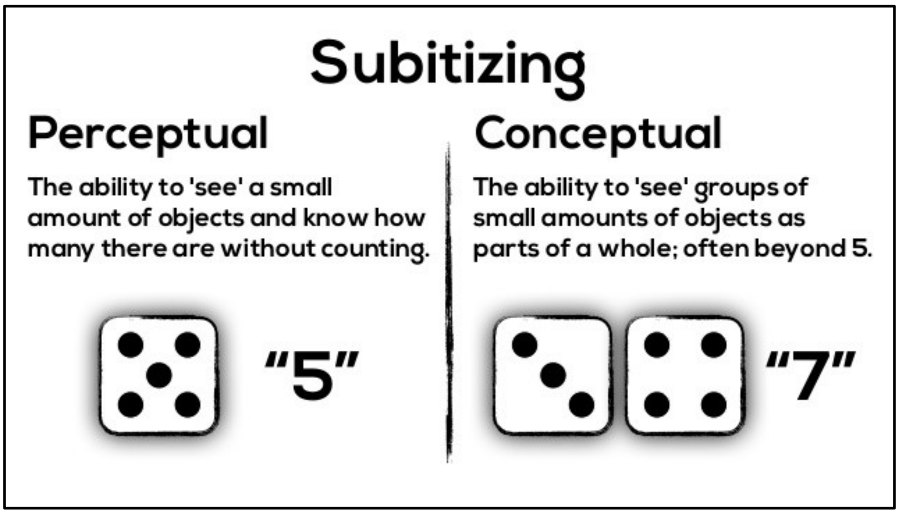


Figure 8. Types of subitizing
(https://images.app.goo.gl/ayTgM2WwttNiXzv36)

The development of the concept of the number in young children began in the 1980s by the habituation method. Prentice Starkey and Robert G. Cooper (1980) studied infants aged 4-7 months. The babies sat on their mother's lap and watched a screen. The researchers looked at how long babies looked at projected images showing two black dots at different distances. After the children lost interest in the two-dot images, three dots suddenly appeared on the screen. This image was viewed by infants for significantly longer periods than the previous two dots. Thus, the three-point image was perceived as being different from the two-point image, which in turn was perceived as similar by infants (Feigenson, L. – Carey, S. – Spelke, E., 2002)

The validity of the results of the above experiments was questioned by other researchers in the late 1990. They claimed that it is almost impossible to design stimuli where only the number of elements is different in the two figures. Certain perceptual variables always change with cardinality. For example, when the number of elements changes, the total circumference of the elements, the area they fill, or the amount of light they reflect also changes. If researchers are interested in the number discrimination abilities of babies, they need to be sure that infants are responding to the cardinality rather than the perceptual variables that correlate with it (Clearfield-Mix, 1999). Most of the studies conducted after 1999 have attempted to control the effects of perceptual variables in many ways and have produced much more reliable results.

Comprehensive questions:

- What are the four innate numeric abilities by D. C., Geary?
- Could you mention three situations when children use subitizing ability in their activities?

5. Mathematical competences in early childhood

Competence is defined by the ability to apply effectively in a variety of situations. It is based on knowledge, skills, experience, values and attitudes. But can we talk about the foundation and development of mathematical competence in early childhood? The basic purpose of mathematical education at all ages is to enrich and shape a child's personality and thinking. In accordance with the age-specific characteristics, playful activities, adherence to the principle of graduality, and the application of experience-based methods of cognition can bring mathematics, as a discipline, to the world of children who live in unity and wholeness. Mathematics can be discovered in the natural and social environment surrounding the child. With the help of appropriate methods, in the kindergarten they can begin to develop the ability to independently acquire knowledge, to develop problem-solving skills, creative thinking, to prepare, to base the number and operation concept, and to calculate skills. The complex view that mathematics is not only a stand-alone science but also a contributor to other sciences, part of our daily lives, part of humanity's cultural heritage, way of thinking, creative activity, the source of the joy of thinking and the representation of order and aesthetics in structures can be effectively grounded in samples. Thus, in the light of the above thoughts, we can safely answer yes to the question whether we can talk about the foundation and development of mathematical competences in kindergarten.

The three components of mathematical competence are mathematical knowledge, mathematical-specific skills, and abilities, and mathematical motivations and attitudes. Obviously, the importance of these three components is different in mathematical education. The acquisition and teaching of abstract mathematical knowledge and scientific concepts can no longer be the goal of early childhood education. Though it cannot be doubted that such activities are also present in children's activities, mostly indirectly. (e.g., knowledge of the circle, concept of the circle during the round games). The most important ability and skill components of mathematical competence are summarized in the following table.

Table 1. The most important skills and competence components of mathematical competence (Source: Fábián et al., 2008)

Skills	Thinking skills	Communication skills	Knowledge acquisition abilities	Learning abilities
• counting • calculation • quantitative inference • estimation • measurement • units of measurement • text task solution	• organizing • combinativity • deductive inference • Inductive inference • probability inference • reasoning • Proving	• relation vocabulary • comprehension • text interpretation • spatial vision, spatial relations • representation • presentation	• problem sensitivity (questions) • problem representation • originality, creativity • problem solving • metacognition	• attention • partial-whole perception • memory • task management • problem solving speed

One of the most important skills in mathematical competence is the ability to think, but it can be realized at the same time through a variety of abilities (e.g., systematization, combinativity, deductive and inductive inference, reasoning). Thus, the ability to think that was developed in kindergarten activities, should become applicable in many other areas of life.

Organizational ability means collecting and systematizing the information and data appearing in the task and the problem raised, and, the ability to integrate the newly acquired knowledge into the system of previous knowledge. The meaning of age-appropriate language development, comprehension, text interpretation, and relational vocabulary does not need to be interpreted, but it must be emphasized that its existence is indispensable for the recognition and understanding of mathematical texts. It is important whether the child has already acquired a knowledge at a skill level and, for example, can he / she solve a problem with on this knowledge in the head, and the amount of memory will play a role. A note of an action (formula) (during applications) indicates the child's associative memory. Intelligent memory can help you learn by understanding the relationships between things to remember. The most important component of early childhood education is the third component, that is, the formation of mathematical-related motives and attitudes, the maintenance of curiosity about mathematical content and experiences hidden in the outside world, the nurturing of interest and inner motivation by games are required (Skemp, 2005).

Mathematical education plays an important role in practicing thought activities, increasing the flexibility of thinking, developing constructive ability and creativity. According to Zoltán Dienes, an internationally renowned Hungarian mathematical professor (2014), children forget most of the mathematics they have learned, so we cannot simply aim at acquiring knowledge. The natural process of maturation and development should not be hastened but enriched. By the age of three, the combined sensory-motor ability of a healthy infant's perception and movement makes him able to handle his or her environment, discover himself or herself in his living space, discover and experience human, natural and material environment.

6. From lecture method to discovery approach

Teaching is much more than transmitting knowledge, facts, information or data. For it is said that: *"A poor teacher tells; An average teacher informs; A good teacher teaches; An excellent teacher inspires"*[15] The new instructional pedagogy requires teachers to move away from lecturing and move towards activity based learning. Most teachers often teach Maths by the "lecture method" even at early years. While teachers becomes very "active", children are rather "passive". This does not lead to a lasting learning. Active learning involves those strategies where children act, touch, feel, participate, discover facts and ideas in the learning process (Azuka, 2013).

Through activity and interest, children will gain experiences of formal, quantitative and spatial relationships of the narrower and broader nature-human-material environment. In the course of discovering reality, they develop a positive emotional relationship with nature and human creations, and they learn to protect them and to preserve values. While discovering the environment, children acquire experience and knowledge of mathematical content and use them in their activities. Teachers should provide opportunity, time, place and tools to gain spontaneous and organized experience and knowledge. They should promote children's independent opinions, the development of decision-making abilities.

Discovery approach of teaching is a method where children are guided by the teacher to discover mathematical facts and formulae through observations and organized activities. In this approach the teacher provides the materials and guides children to carry out some activities which would lead them to arrive at a new knowledge. Discovery activities could be done individually or in groups of few children. This approach enables children to actively participate in the learning process and discover things for themselves.

[15] URL: https://issuu.com/alexanderdecker/docs/activity_-based_learning_strategies [2021.06.16.]

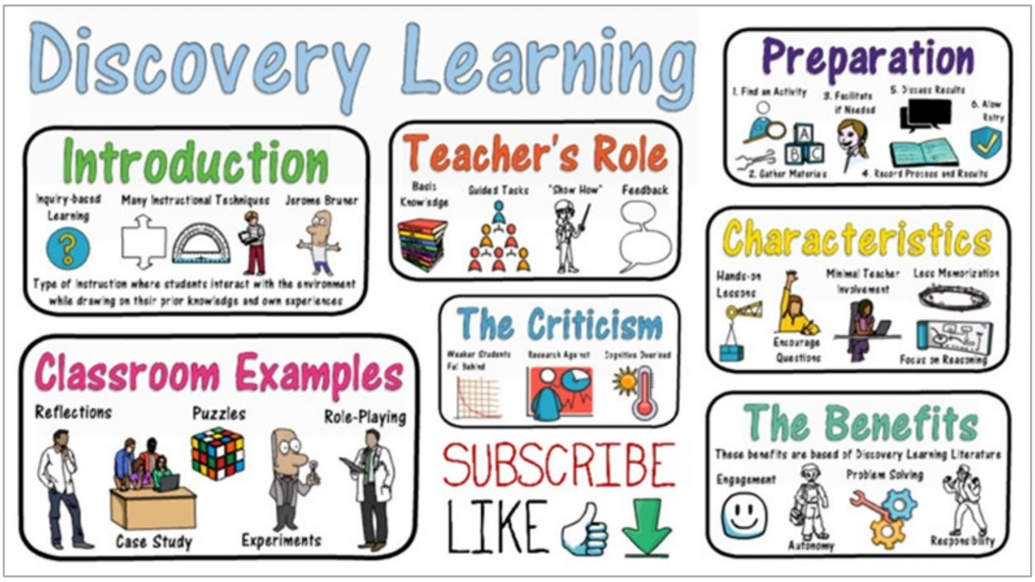


Figure 9. Discovery approach of learning
(https://youtu.be/i6j1YxxbogM)

For most children play and work-like activities provide the most effective learning by which understanding of mathematics and sciences can develop. When a subject is presented as a mass of sheer facts, children are not able to form patterns and establish meaningful relations among the stimuli, or connect them with their own past experiences. The result is a distorted concept formation and a distressing tendency to avoid the subject later in life (Elliot, Thomas & Joan, 2000). This idea is also underlined by the ancient proverb that states: *"what I hear I forget; what I see I remember; what I do I understand"*. This means that until a child practicalises a concept or participates in the learning process, he cannot understand the concept. To evaluate teaching is to evaluate the extent children have been inspired to think and create ideas. This can only be achieved through practical activities which make children to be active in the teaching-learning process.

The following basic mathematical dimensions can be introduced at an early age to enhance a child's understanding of Math through practical activities:

1. *Math Language* – terms such as "more than", "greater than", "less than", "equal to" and so on.
2. *Number Sense* – This is the aspect of Math that outlines facts such as the number 2 represents two objects and that 3 is greater than 2, but less than 5, etc.
3. *Measurements* – various types of measurements (e.g. amounts, sizes, and even distances) that must be taught in Math fundamentals for children.
4. *Geometrics* – Children should learn various shapes, patterns, and individual characteristics and features of objects.
5. *Spatial Relationships* – Children learn about objects that are in front, in back, and to the side. Also, this covers the distance concepts of 'near' and 'far'.
6. *Exploring shapes* − Everything in our world has a unique shape. Examples of shapes (e.g. circle, square, triangle, oval, rectangle, and diamonds) should be taught in basic Math for children.
7. *Sorting* − Sorting by the color, by the shape, by the unique texture associated with the objects, the size, and even the category could be taught at early age. This helps children learn how to sort similar items so that tasks such as counting and dividing become much easier.

8. *Patterning* − taking an item, such as blocks, crayons, and math manipulative and grouping them in such a way that one design is consistent. Patterns can be taught through the integration of music and math games.
9. *Counting* – Encourage children to count objects in their world, such as toys, buttons, blocks, windows, doors, cars, and other items. It does not only enhance their overall knowledge of the number system and mathematics, but also optimize their awareness of things existing around them.[16]

Children are able to understand symbols and abstract concepts only after experiencing the ideas on a concrete level. They need to have Math experiences that incorporate their senses, that require them to experiment and make observations, and that allow them time to investigate a topic further. Children learn Math with concrete materials, thus they need concrete objects like real stuff, manipulatives, materials, such as blocks, counters, popsicle sticks, in order to make sense of new math concepts or abstract ideas. After children had several opportunities to learn a new concept with real objects they are ready to connect their learning to abstract symbols such as numbers and math symbols. However they need plenty of time to play with math materials before they use them for teacher guided Math activities. It is also important linking Math to the everyday experiences. Math games and activities are also very good opportunities to build Math vocabulary, but the general principle is *"things before ideas and ideas before words"* (Azuka, 2013).

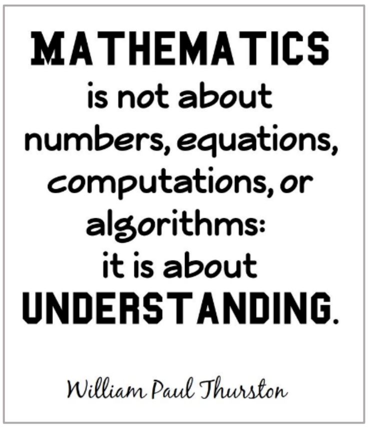
Comprehensive questions:

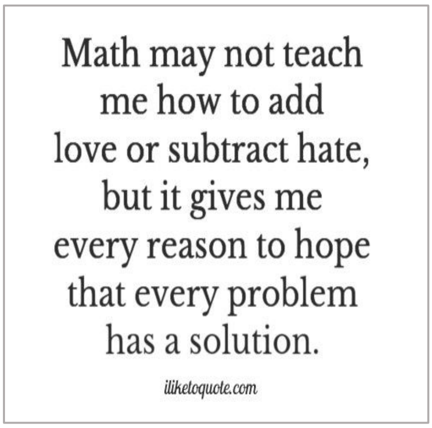
- How would you introduce basic mathematical dimensions (p. 71) by play or by practical activities?
- How could you interpret Azuka's concept into your teaching practice? *"Things before ideas and ideas before words"*

7. Conclusion

Children mathematical knowledge at the early years predicts later academic achievement better than early reading or attention skills. Math is not only measuring, sorting, noticing patterns, making comparisons, counting, but also thinking in structures and solving problems. Therefore we can use many ways to incorporate math learning into the daily practice of teaching. Teachers can foster a positive attitude toward math. They can find ways to incorporate enjoyable math activities and math talk into regular activities like cooking, setting the table, and going for a neighbourhood walk. Discovery approach of teaching strengthens problem solving and using mistakes as an opportunity to promote growth mindset. When children focus on problem solving rather than on getting the right answer they learn more. "*Discovery learning is the method that takes place when a teacher sets up an experiment, acts as a coach, and provide clues along the way to help students come to solutions."* (Krisnawati cited Hedge, 2003)

[16] URL: https://kiwipreschool.com/santa-rosa/child-care/teaching-basic-math [2021. 02. 16.]





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