# Stand structure and growth of mixed white poplar (Populus alba L.) and black locust (Robinia pseudoacacia L.) plantations in Hungary 

Károly RÉdeI $^{\text {a1 }}$ - Irina VEPERDI ${ }^{\text {a }}$ - Henrik Meilby ${ }^{\text {b }}$<br>${ }^{a}$ Hungarian Forest Research Institute, Budapest, Hungary<br>${ }^{\mathrm{b}}$ The Royal Veterinary and Agricultural University, Danish Centre for Forest, Landscape and Planning, Denmark


#### Abstract

The paper deals with the stand structure and yield of black locust (Robinia pseudoacacia L.) plantations mixed with white poplar (Populus alba L.) in various proportions, partly applying a new methodological approach. The main stand structure and yield factors were determined separately for each species, measured stem by stem, using volume functions prepared for each species. To demonstrate the advantage of white poplar - black locust mixed plantations over monocultures, a set of models expressing single-tree growth was elaborated. The models were applied to simulated stands with different proportions of $P$. alba and $R$. pseudoacacia. The estimated growth indicated that the advantage of mixing the two species peaked at about $40-50 \%$ P. alba (basal area percentage) between age 16 and 21 . The trial may indicate that if two species have fast initial growth rates and similar rotation ages, planting them in mixed stands might lead to increased yield.


## Forest yield / P. alba and R. pseudoacacia mixed stands / single-tree growth model


#### Abstract

Kivonat - Fehér nyárral (Populus alba L.) elegyes akácosok (Robinia pseudoacacia L.) faállományszerkezete és fatermése Magyarországon. A tanulmány a fehér nyárral (Populus alba L.) elegyes akácosok (Robinia pseudoacacia L.) faállomány-szerkezetét és fatermését vizsgálja különböző elegyvariációkban részben új módszertani megközelítéssel. A főbb állományszerkezeti és fatermési tényezőket fafajonként külön-külön, törzsenkénti felvétellel, az egyes fafajokra kidolgozott fatérfogatfüggvény alapján határoztuk meg. A fehér nyárral elegyes akácosok termesztése előnyös voltának bemutatására egyes fa növekedési modelleket dolgoztunk ki, amelyek segítségével a fehér nyár és az akác különböző elegyvariációit vizsgáltuk. A növekedés vizsgálatok azt jelezték, hogy az említett fafajok elegyes termesztése a fehér nyár 40-50\%-os körlapösszeg szerinti elegyaránya mellett a legelőnyösebb 1621 éves kor között. A kísérlet azt is bizonyítja, hogy ha két fafajnak gyors kezdeti növekedési erélye van és hasonló a vágásérettségi koruk, akkor elegyben telepítésük esetén nagyobb fatermés érhetỏ el.


## Fatermés / fehér nyár elegyes akácosok / egyes fa növekedési modell

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## 1 INTRODUCTION

Technical guidelines of primary forest production are usually aimed at the establishment and management of pure stands. However, mixed stands may provide advantages that justify mixed cultivation of two or three species, provided that the site conditions are suitable. Mixing has mainly biological and ecological advantages, but it may also have a positive effect on yield. In addition, non-material benefits of mixed forests, e.g. improvement of the environment and satisfaction of social expectations, are increasingly acknowledged. The scientific study of mixed forests is a more complex task than the study of pure stands. This partly explains why only limited efforts have been made so far to quantify the growth and yield of mixed stands and to compare the results with pure stands.

Forestry professionals started the description of the structure and species composition of mixed forests by the end of the 18th century. Oak was studied first, followed by the admixed tree species. Later other mixed forest types were studied. In the beginning of the 19th century mainly monoculture forests were established but from the middle of the century there was a shift back to mixed forests (Heyer 1854).

Early in the 20th century analyses made in Central Europe showed that the volume production of mixed spruce and Scots pine stands exceeded that of pure stands (Schilling 1925, Busse 1931). Similarly, it has been observed that the admixing of birch in spruce forests does not reduce the yield of spruce (Fiedler 1966). For instance, a mixture of $40 \%$ birch and $60 \%$ spruce provided a greater total yield than the same proportions of these species when grown in pure stands (Lappi-Seppala 1930). Furthermore, at fertile sites Wiedemann (1943) reported that the yield of mixed beech and spruce forests was higher than that of pure spruce stands. By contrast, at poor sites the yield of mixed stands was lower than that of pure spruce stands.

Assmann (1970) paid great attention to the study of mixed stands in his book, 'The Principles of Forest Yield Study'. Although it is almost impossible to define comparable pure and mixed experimental plots (for a given set of species), the main finding of the author is that mixing generally results in a greater total yield.

In practice the choice between mixed or pure stands is a complex issue. Mixed stands are often more resistant to pests and have a higher production level than pure ones. In most cases, the total yield of mixed stands exceeds that of light-demanding tree species grown in monoculture (Smith 1986). On the other hand, a balanced development of mixed stands is only obtained when the competition between the species involved is not too asymmetric. This implies that the species composition of mixed stands and the thinning regime should be adjusted to the site conditions but, as attempts to do this are not always successful, mixed stands are frequently claimed by foresters to be more difficult to control than pure stands.

Traditionally, simple stand growth models (growth and yield tables) have been used to predict the development of pure, even-aged stands. However, due to the higher complexity of mixed stands this approach is not generally applicable in this case. Instead the development of mixed stands is usually modelled at the diameter-class or single-tree level. For instance, single-tree growth models have been developed for mixed broad-leaved forests of the Appalachian mountains (Harrison et al. 1986), for coniferous forests in the North Western United States (Wykoff 1990), for mixed stands of spruce and pine in Finland (Pukkala et al. 1994,1998), and for yield forecasting in thinned birch-spruce stands in Sweden (Tham 1988,1989). Application of spatially explicit models like these implies that the stand growth simulation is based on the spatial location and initial diameter of the trees. The models include relationships between diameter and height and competition indices describing the specific growth conditions of each individual tree.

Two recent volumes summarise the latest research work including results on modelling the structure and yield of mixed stands (Robertson, Cannell and Malcolm 1992; Costa and Preuhsler 1994). Based on the literature it appears that two distinct directions of research can be identified. One direction is concerned with the extension of empirical growth and yield models already prepared for pure stands to mixed ones. The other direction focuses on dynamic models based on the physiology of individual trees. In the present context simple empirical models are applied.

The objective of this study was to examine the stand structure and growth in an experiment with mixed white poplar and black locust plantations in Hungary. Particularly, the observations from the experiment are used as a basis of estimating the potential advantage of mixed stands and the optimal basal area share of each of the two species. In this respect we considered also some methodological tools improved by Hungarian researchers (Béky Somogyi 1995, 1999, Solymos - Béky 1995, Rédei 1999).

## 2 MATERIAL

Black locust is frequently planted together with white poplar on the sandy ridges between the Danube and Tisza rivers in Central Hungary. In forests covering more than 14,000 ha in this region the joint share of black locust and white poplar exceeds $80 \%$, and $70 \%$ of the stands are less than 20 years old.

The experiment examined in this paper is located in the Danube-Tisza Interflow region, in the central part of Hungary (location: Ballószög, altitude: less than 80 m ; latitude: N 46042 '; longitude: E $19040^{\prime}$ ). The trial was established in 1995 in a 16 -year-old stand that was planted in spring 1979 in an agricultural field with one-year old seedlings after shallow ploughing. The initial proportions of the tree species in scattered mixture were $70 \%$ black locust and $30 \%$ white poplar. According to the Hungarian classification of forest site types, the study area is located in the forest steppe climate zone with a relative air humidity of less than $50 \%$ in July at 2 pm and an annual precipitation of less than 550 mm . The soil is humic sand with a very shallow rootable depth ( $<20 \mathrm{~cm}$ ).

Five plots were established within sub-compartment Kecskemét-Ballószög 20 C , in which black locust and white poplar occurred in various proportions. The area of plots $1-4$ was 0.1 ha each, whereas that of plot 5 was only 0.035 ha . The breast height diameter and $\mathrm{x}, \mathrm{y}$ coordinates of all trees were measured. The crown diameters were measured in four directions corresponding to the four points of the compass. Finally, height was measured for 20 per cent of the trees in plots $1-4$ and 30 per cent of the trees in plot 5 . All trees in the five plots were classified according to the tree height classification system generally used in Hungarian silvicultural practice: $1=$ dominant tree, $2=$ co-dominant tree, $3=$ intermediate (partly dominated) tree, $4=$ suppressed tree. The plots were measured again in 2000 at age 21.

The spatial structure and diameter distributions of the plots are shown in Figure 1. As visible in the Figure the experiment includes plots with very different shares of the two species as well as plots with more even shares.

## 3 METHODS

The main stand structure and yield statistics were determined separately for each species at ages 16 and 21 years (1995 and 2000). Volumes were calculated on the basis of stem-by-stem estimates. First, semi-logarithmic diameter-height regressions for each species were prepared.

Subsequently, these regressions were used for estimating the height of each tree. Finally, volumes were estimated using the volume functions developed by Sopp (1974):
for white poplar:

$$
v=10^{-8} d^{2} h^{1}(h /[h-1.3])^{2}(-0.4236 d h+12.43 d+4.6 h+3298),
$$

for black locust:

$$
v=10^{-8} d^{2} h^{1}(h /[h-1.3])^{2}(-0.6326 d h+20.23 d+3034)
$$

where $v$ is stem volume $\left(\mathrm{m}^{3}\right)$,
$d$ is diameter at breast height ( cm ), and
$h$ is tree height (m).
The experiment does not include replications and is poorly balanced (cf. Figure 1). Therefore, we cannot evaluate the advantage of mixed plantations by analysis of variance. However, as will appear from Figure 1 the plots include a wide range of competition states for both species and, hence, the advantage of mixed plantations with white poplar and black locust can perhaps be evaluated on the basis of the variation of growth observed for individual trees. The procedure includes the following steps: (i) elaborate a set of single-tree growth equations utilising the spatial structure of the plots and taking site-quality variation into account; (ii) hypothesize various stem numbers and species compositions and create hypothetical stands; (iii) use the growth models to evaluate the development of the hypothetical stands assuming homogeneous site quality; (iv) for each hypothetical stand and species calculate the deviation of the predicted volume production from that of a pure stand with the same stem number and share of the area; (v) use the results as a basis of estimating the optimal species composition at the particular site for the age interval 16-21.

The single-tree model should include one or more variables describing the competition experienced by each tree. One such variable is the current diameter of the tree, but including additional variables that describe the immediate neighbourhood of the tree may improve the model. A wide variety of such competition indices have been tried out and in unthinned stands it appears that, no matter which competition index is used, the explanatory power of such indices remains relatively low when the present size of the tree is also included in the model (e.g. Wichmann 2002).

To simplify matters it was decided to use the local basal area (including the tree itself) within a circle with radius 5 metres as the basic competition index. The index was calculated for each of the two species ( $P$. alba: $G_{P} ; R$. pseudoacacia: $G_{R}$ ) and in total for both species $\left(G_{T}\right)$ and, as it appeared unlikely that the competition effects of $P$. alba and R. pseudoacacia were identical, the percentages of the total, local basal area were also calculated for each species ( $P$. alba: $P_{P} ;$ R. pseudoacacia: $P_{R}$ ).

A range of different models including the above competition indices were tested and on the basis of the standard deviation and distribution of the residuals the following two models were chosen to be applied for growth prediction (iii):

$$
\begin{gather*}
\Delta d_{i j k}=\alpha_{0}+\alpha_{1 i}+\alpha_{2 j}+\left(\beta_{0}+\beta_{1 j}\right) d_{i j k}+\left(\gamma_{0}+\gamma_{1 i}\right) G_{T, i j k}+\left(\delta_{0}+\delta_{1 j}\right) P_{P, i j k}+\varepsilon_{i j k}  \tag{1}\\
\Delta v_{i j k}=\alpha_{0}+\alpha_{1 i}+\alpha_{2 j}+\left(\beta_{0}+\beta_{1 i}\right) d_{i j k}+\left(\gamma_{0}+\gamma_{1 j}\right) d_{i j k}^{2}+\delta_{0} G_{T, i j k}+\left(\eta_{0}+\eta_{1 j}\right) P_{P, i j k}+\varepsilon_{i j k} \tag{2}
\end{gather*}
$$

where: $G_{T}$ is local basal area (both species) within a circle with radius 5 metres ( $\mathrm{m}^{2} / \mathrm{ha}$ ),
$G_{T}=G_{P}+G_{R} ;$
$P_{P} \quad$ is contribution of $P$. alba to the local basal area (\%), $P_{P}=100 \% \times G_{P} / G_{T}$;
$i \quad$ is the plot number, $i=1 \ldots 5$;
$j \quad$ is the species, $[P$. alba; R. pseudoacacia];
$k \quad$ is the tree, $k=1 . . . n$;
$\Delta d_{i j k}$ is diameter growth ( $\mathrm{cm} / \mathrm{year}$ );
$\Delta v_{i j k}$ is volume growth ( $\mathrm{m}^{3} /$ year); and
$\varepsilon_{i j k}$ are independent and normally distributed error terms.
Diameters and competition indices were measured in 1995.


Figure 1. TOP: Stem maps of the experimental plots (Kecskemét-Ballószög 20 C). Symbol diameter is proportional to breast height diameter in 1995 (age 16). Bоттом: Diameter distributions at ages 16 (1995) and 21 (2000)

## 4 RESULTS

We started by testing the effects of plot and species on the relationship between diameter ( cm ) at breast height and height (m) in each of the years, 1995 and 2000. In 1995 it turned out that there was an almost significant effect of plot ( $\mathrm{P}=0.06$ ), and in 2000 there was a significant effect of species $(\mathrm{P}=0.01)$. We therefore decided to continue with separate models for each year and species, testing whether significant effects of plot remained. Except for Robinia pseudoacacia in 1995 ( $\mathrm{P}=0.06$ ) it emerged that this was not the case. Accordingly, we used the following four models to estimate tree height:
P. alba, 1995:

$$
\begin{aligned}
h & =-3.29+6.81 \ln (d) \\
h & =-3.26+6.94 \ln (d) \\
h & =-5.44+8.72 \ln (d) \\
h & =-5.58+8.28 \ln (d)
\end{aligned}
$$

RMSE $=1.16 \mathrm{~m}$
RMSE $=1.57 \mathrm{~m}$
RMSE $=1.96 \mathrm{~m}$
RMSE $=2.02 \mathrm{~m}$

Summary statistics regarding the stand structure and increment in the period 1995-2000 are shown in Table 1 for each species and plot, and in total for each plot. It appears that the initial stem number per hectare (1995) is almost similar for all plots (1420-1600/ha). The share of $P$. alba stems ranges from 4 to 82 percent (cf. Fig. 1). As for the stand diameter (expressed as $\mathrm{D}_{\mathrm{g}}$ ) it immediately appears that for $R$. pseudoacacia $\mathrm{D}_{\mathrm{g}}$ is $12-13 \mathrm{~cm}$ on plots $2-5$ where the number of R. pseudoacacia stems is relatively high, whereas on plot 1 it is 14.2 cm . Similarly, the $\mathrm{D}_{\mathrm{g}}$ of $P$. alba is $14-15 \mathrm{~cm}$ on plots $1-3$, whereas it is $19-23 \mathrm{~cm}$ on plots $4-5$. Similar observations can be done as regards the variation of stand height $\left(\mathrm{H}_{\mathrm{g}}\right)$. Thus, it appears that when the share of a species is sufficiently low the trees are generally larger, both as regards diameter and height. The diameter increment of $P$. alba is almost twice that of $R$. pseudoacacia on plots $1-3$, and on plots $4-5$ the difference becomes even greater as the few $P$. alba trees are mostly large-diameter trees. On plots $4-5$ the diameter increment of the $R$. pseudoacacia trees appears slightly reduced.

As regards the volume in 1995, plots 3-5 are almost similar whereas the volume of plots 1 and 2 is slightly greater. As expected the volume increment of $P$. alba is generally higher than that of R. pseudoacacia and, therefore, the total volume increment of a plot appears to increase with increasing share of $P$. alba. However, the volume increments of plots 3 and 4 are similar although their shares of P. alba and R. pseudoacacia differ markedly. Accordingly, the variation of volume increment between plots cannot be explained as a consequence of species composition alone. There was no possibility to include effects of potential microsite differences. Random variation due to the small plots, variation of growth conditions between plots, and variation as regards diameter class distribution and spatial distribution of trees must be involved as well. Obviously this would not be a problem had the experiment included a sufficient number of replications. Therefore an analysis at the single-tree (or sub-plot) level is needed.

Table 1. Summary statistics regarding stand structure and increment in the five experimental plots of subcompartment Kecskemét-Ballószög 20 C.

| Plot Species | $\begin{gathered} \hline \mathrm{D}_{\mathrm{g}} \\ 1995 \end{gathered}$ | $\begin{gathered} \hline \mathrm{D}_{\mathrm{g}} \\ 2000 \end{gathered}$ | $\mathrm{i}_{\text {Dg }}$ | $\begin{gathered} \mathrm{H}_{\mathrm{g}} \\ 1995 \end{gathered}$ | $\begin{gathered} \hline \mathrm{H}_{\mathrm{g}} \\ 2000 \end{gathered}$ | $\mathrm{i}_{\mathrm{Hg}}$ | $\begin{gathered} \mathrm{N} \\ 1995 \end{gathered}$ | $\begin{gathered} \mathrm{N} \\ 2000 \end{gathered}$ | $\mathrm{l}_{\mathrm{N}}$ | $\begin{gathered} \mathrm{G} \\ 1995 \end{gathered}$ | $\begin{gathered} \mathrm{G} \\ 2000 \end{gathered}$ | $\mathrm{i}_{\text {G }}$ | $\begin{gathered} \hline \mathrm{V} \\ 1995 \end{gathered}$ | $\begin{gathered} \hline \mathrm{V} \\ 2000 \end{gathered}$ | $\mathrm{i}_{\mathrm{V}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | cm |  | cm/yr | m |  | $\mathrm{m} / \mathrm{yr}$ | /ha |  | ha/yr | $\mathrm{m}^{2} / \mathrm{ha}$ |  | $\mathrm{m}^{2} \mathrm{ha} / \mathrm{yr}$ | $\mathrm{m}^{3} / \mathrm{ha}$ |  | $\mathrm{m}^{3} \mathrm{ha} / \mathrm{yr}$ |
| 1 Populus | 14.8 | 17.0 | 0.45 | 15.1 | 19.3 | 0.85 | 1320 | 1300 | -4 | 22.60 | 29.56 | 1.390 | 185.9 | 302.6 | 23.33 |
| 1 Robinia | 14.2 | 15.6 | 0.27 | 15.2 | 17.2 | 0.40 | 280 | 280 | 0 | 4.46 | 5.35 | 0.179 | 41.0 | 53.3 | 2.48 |
| 1 |  |  |  |  |  |  | 1600 | 1580 | -4 | 27.06 | 34.91 | 1.569 | 226.9 | 355.9 | 25.81 |
| 2 Populus | 15.0 | 17.8 | . 56 | 15.2 | 19.7 | . 90 | 980 | 900 | -16 | 17.38 | 22.40 | 1.004 | 145.1 | 234.6 | 17.89 |
| 2 Robinia | 12.2 | 13.7 | 0.30 | 14.1 | 16.1 | 0.40 | 560 | 560 | 0 | 6.58 | 8.28 | 0.340 | 57.5 | 78.7 | 4.23 |
| 2 |  |  |  |  |  |  | 1540 | 1460 | -16 | 23.96 | 30.68 | 1.344 | 202.6 | 313.2 | 22.12 |
| 3 Populus | 14.5 | 17.1 | 0.51 | 14.9 | 9.3 | 0.88 | 950 | 910 | -8 | 15.70 | 20.77 | 1.014 | 128.3 | 213.3 | 16.99 |
| 3 Robinia | 12.6 | 14.1 | 0.30 | 14.3 | 16.3 | 0.40 | 590 | 580 | -2 | 7.36 | 9.04 | 0.336 | 64.7 | 86.3 | 4.32 |
| 3 |  |  |  |  |  |  | 1540 | 1490 | -10 | 23.05 | 29.80 | 1.350 | 193.0 | 299.6 | 21.31 |
| 4 Populus | 19.2 | 24.2 | 0.99 | 16.8 | 22.3 | 1.10 | 270 | 270 | 0 | 7.81 | 12.37 | 0.912 | 70.0 | 141.6 | 14.32 |
| 4 Robinia | 12.2 | 13.4 | 0.24 | 14.1 | 15.9 | 0.36 | 1150 | 1150 | 0 | 13.47 | 16.19 | 0.543 | 115.9 | 150.2 | 6.87 |
| 4 |  |  |  |  |  |  | 1420 | 1420 | 0 | 21.28 | 28.56 | 1.455 | 185.9 | 291.8 | 21.19 |
| 5 Populus | 23.0 | 29.8 | 1.37 | 18.1 | 24.2 | 1.22 | 57 | 57 | 0 | 2.36 | 3.97 | 0.322 | 23.5 | 50.9 | 5.49 |
| 5 Robinia | 12.6 | 13.8 | 0.23 | 14.3 | 16.1 | 0.36 | 1453 | 1453 | 0 | 18.15 | 21.63 | 0.697 | 159.9 | 205.8 | 9.17 |
| 5 |  |  |  |  |  |  | 1510 | 1510 | 0 | 20.51 | 25.60 | 1.019 | 183.4 | 256.7 | 14.65 |

As a first attempt to estimate the advantage of mixed plantations of white poplar and black locust over pure plantations of these species, the volume increments were calculated for $250 \mathrm{~m}^{2}$ sub-plots $(12.5 \times 20 \mathrm{~m})$, i.e. one fourth of the plots $(25 \times 40 \mathrm{~m})$. The volume increment of each species ( $i_{V, P}, i_{V, R}$ ) was plotted against its contribution to the basal area of the sub-plot $\left(P_{P}, P_{R}\right)$. If growth in mixed populations is greater than in pure stands of the same species, a non-linear (concave) variation would be expected. For $P$. alba no deviation from linearity could be observed but for R. pseudoacacia a concave relationship was detected and the power
$(\theta=0.70)$ of the relationship $i_{V R}=\phi P_{R}^{\theta} \Leftrightarrow \ln \left(i_{V R}\right)=\ln (\phi)+\theta \ln \left(P_{R}\right)$ was significantly different from $1(\mathrm{P}>\mathrm{F}=0.004)$.

## Growth models and simulation

As the next step towards determination of the advantage of mixing the two species we estimated the parameters of the single-tree models, (1) and (2). The results are presented in Table 2. The differences between plots are considerable. Moreover, it appears that the growth of white poplar varies much more with tree diameter than that of black locust. Finally, the fact that $\delta_{1 \mathrm{j}}$ and $\eta_{1 \mathrm{j}}$ are negative for white poplar, and numerically greater than the positive $\delta_{0}$ and $\eta_{0}$ parameters, implies that an increasing basal area share of white poplar leads to reduced growth of this species. On the other hand, the growth of black locust is enhanced. Similarly, a reduced basal area share of white poplar (increased share of black locust) leads to increased growth of white poplar and reduced growth of black locust. Accordingly, for both species the observed increment of trees growing in mixed neighbourhoods was greater than that of trees growing in pure neighbourhoods.

Table 2. Estimated parameters of the two single-tree models applied for growth prediction. Standard errors are given in parentheses. Units of measurement: $\mathrm{d}: \mathrm{cm}, G_{T}: \mathrm{m}^{2} / \mathrm{ha}$, $P_{p}$ : per cent, $\Delta d$ : cm/year, $\Delta v: m^{3} /$ year .

| Diameter increment model (1) |  |  |  | Volume increment model (2) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RMSE $=0.1964$ | $\mathrm{R}^{2}=0.6620$ |  |  | $\mathrm{RMSE}=0.007568 \quad \mathrm{R}^{2}=0.8836$ |  |  |  |  |
|  | Plot | $\alpha_{1 i}$ | $\gamma_{1 \mathrm{i}}$ |  |  | Plot | $\alpha_{1 i}$ | $\beta_{1 \mathrm{i}}$ |
| $\begin{array}{cc}  & 0.117162 \\ \alpha_{0} & (0.0965) \end{array}$ | 1 | $\begin{gathered} -0.177632 \\ (0.1180) \end{gathered}$ | $\begin{gathered} 0.005790 \\ (0.0048) \end{gathered}$ | $\alpha_{0}$ | $\begin{gathered} -0.000823 \\ (0.0035) \end{gathered}$ | 1 | $\begin{gathered} 0.016758 \\ (0.0038) \end{gathered}$ | $\begin{gathered} -0.001426 \\ (0.0003) \end{gathered}$ |
| $\begin{array}{lc} \beta_{0} & 0.014464 \\ (0.0028) \end{array}$ | 2 | $\begin{gathered} 0.076948 \\ (0.1122) \end{gathered}$ | $\begin{gathered} -0.004714 \\ (0.0047) \end{gathered}$ |  | $\begin{gathered} -0.000305 \\ (0.0004) \end{gathered}$ | 2 | $\begin{gathered} 0.012030 \\ (0.0037) \end{gathered}$ | $\begin{gathered} -0.001167 \\ (0.0003) \end{gathered}$ |
| $\begin{array}{lc} \gamma_{0} & -0.003970 \\ (0.0043) \end{array}$ | 3 | $\begin{gathered} -0.067341 \\ (0.1116) \end{gathered}$ | $\begin{gathered} 0.001677 \\ (0.0047) \end{gathered}$ |  | $\begin{aligned} & 0.000082 \\ & (0.00001) \end{aligned}$ | 3 | $\begin{gathered} 0.009232 \\ (0.0037) \end{gathered}$ | $\begin{gathered} -0.000887 \\ (0.0003) \end{gathered}$ |
| $\begin{array}{ll} \delta_{0} & 0.001258 \\ (0.0006) \end{array}$ | 4 | $\begin{gathered} 0.016709 \\ (0.1007) \end{gathered}$ | $\begin{gathered} -0.000347 \\ (0.0046) \end{gathered}$ |  | $\begin{aligned} & -0.000109 \\ & (0.00004) \end{aligned}$ | 4 | $\begin{gathered} 0.001631 \\ (0.0034) \end{gathered}$ | $\begin{gathered} -0.000154 \\ (0.0003) \end{gathered}$ |
|  | 5 | $\begin{gathered} 0 \\ (-) \end{gathered}$ | $\begin{gathered} 0 \\ (-) \end{gathered}$ |  | $\begin{aligned} & 0.000035 \\ & (0.00002) \end{aligned}$ | 5 | $\begin{gathered} 0 \\ (-) \end{gathered}$ | $\begin{gathered} 0 \\ (-) \end{gathered}$ |
| Species | $\alpha_{2 j}$ | $\beta_{1 \mathrm{j}}$ | $\delta_{1 j}$ | Species |  | $\alpha_{2 j}$ | $\gamma_{1 j}$ | $\eta_{1 \mathrm{j}}$ |
| P. alba | $\begin{gathered} -0.370410 \\ (0.0747) \end{gathered}$ | $\begin{gathered} 0.054722 \\ (0.0037) \end{gathered}$ | $\begin{gathered} -0.004120 \\ (0.0008) \end{gathered}$ | P. alba |  | $\begin{gathered} -0.003759 \\ (0.0025) \end{gathered}$ | $\begin{gathered} 0.000109 \\ (0.000006) \end{gathered}$ | $\begin{aligned} & -0.000137 \\ & (0.00003) \end{aligned}$ |
| R. pseudoacacia | $\begin{gathered} 0 \\ (-) \\ \hline \end{gathered}$ | $\begin{gathered} 0 \\ (-) \\ \hline \end{gathered}$ | $\begin{gathered} 0 \\ (-) \\ \hline \end{gathered}$ | R. pseudoacacia |  | $\begin{gathered} 0 \\ (-) \\ \hline \end{gathered}$ | $\begin{gathered} 0 \\ (-) \\ \hline \end{gathered}$ | $\begin{gathered} 0 \\ (-) \\ \hline \end{gathered}$ |

To get a basic idea about the effect of mixing the two species model (2) can be applied for particular values of diameter, total basal area $\left(G_{T}\right)$ and percentage of $P$. alba $\left(P_{P}\right)$. The resulting volume growth can be compared with predicted growth for pure stands. Not surprisingly, it emerges that the estimated advantage of mixing the species depends on the chosen values of diameter and stand basal area. Therefore, to quantify the apparent advantage of mixed stands in Kecskemét-Ballószög 20 C (1995-2000) it is necessary to consider the growth of stand structures with properties similar to those observed in the plots, i.e. structures with similar diameter distributions, densities, and spatial distributions of trees. Therefore, models (1) and (2) were incorporated into a simulation program and used to evaluate the growth of mixed stands for a range of stem numbers and species compositions. For each combination of stem number and species composition the program generated a stand with diameter distribution and tree heights similar to those observed within the five plots in 1995. Thus, for white poplar the applied mean and standard deviation of diameter were 14.43 and
4.82 cm respectively, and for black locust these were 11.91 and 3.98 cm , respectively. The simulated tree growth was stochastic with mean corresponding to the predictions of models (1) and (2) for plot 5 and variance corresponding to the calculated RMSE (Table 2). The spatial distribution of trees was adjusted such that it resembled that of the experimental plots. The area of the simulated stands was 4 ha, i.e. 40 times the area of each of the experimental plots (1-4). Due to the uncertainty of growth caused by stochastic tree growth, variation of tree diameter and height, and variation of the spatial distribution of trees, this was necessary to obtain stable results.

Simulated stands were prepared for stem numbers ranging from 1000 to 2000 per ha and for species mixtures ranging from $0 \%$ to $100 \%$ white poplar (basal area shares). For each combination of stem number and species composition the volume increments were calculated for each of the species. Based on these results the apparent advantage of mixed stands over single-species stands with same stem number and basal area was estimated for the period from age 16 to 21. The results are shown in Figure 2 and as will appear from the Figure, the extra volume increment obtained when mixing the two species reached a maximum for both species at $40-50 \%$ basal area share of white poplar.


Figure 2. Estimated advantage $\left(m^{3} h a^{-1} y r^{-1}\right)$ of mixing white poplar and black locust in various proportions and at different densities. Share of Populus alba: contribution of P. alba to the basal area in 1995.

## 5 DISCUSSION

When creating the simulated stands considerable effort was made to ascertain that the structural properties of the stands were similar to those of the experimental plots. In addition the simulations were based on plots with an area of 4 hectares. Still, it was observed that the results varied somewhat between simulation runs. Therefore, one should be cautious when interpreting the results in Figure 2.

The advantage of mixed stands appeared to increase when the stem number increased from 1000 to 2000 stems per hectare. However, it should be noted that this increase is likely to be a consequence of the fact that the simulation model did not include a mortality
component. This implies that the model would potentially allow us to increase the stem number at age 16 to, e.g. $10,000 \mathrm{ha}^{-1}$ and still produce an increased volume production. Therefore, the main result of the simulation is that for both species the advantage of mixed stands over single-species stands appeared to peak at about $40-50$ per cent $P$. alba. Moreover, in the current case the maximum advantage of mixed stands appeared to correspond to roughly 12-14 percent for $P$. alba and 14-18 percent for R. pseudoacacia. However, as the volume production of R. pseudoacacia was lower than that of $P$. alba the absolute advantage was highest for $P$. alba (cf. Table 1).

The height growth of white poplar is faster than that of black locust when these two species are planted together (cf. Table l) and black locust could never overgrow white poplar in mixed stands. Due to its nitrogen-fixing ability black locust has a stimulating effect also on growth of other tree species growing in the same stand. The difference is particularly impressive when white poplar trees are scattered in a stand and their share of the stems does not exceed $20 \%$ (plots 4 and 5). A similar pattern is observed for radial growth.

The trial appears to indicate that if both species have fast initial growth rates and similar rotation ages, planting them in mixed stands may lead to increased yield. However, planting (mixing) schemes have to be chosen in such a way that they support the compatibility of the two species. In addition, one must realise that the mixing scheme influences the character of intermediate thinnings. For example, if a site is suitable for both black locust and white poplar and they are mixed at the single-tree level, the dominating species with the larger number of stems should be preferred for removals in course of intermediate thinnings. If the two species are planted in groups, the growth rates and the proportion of each species within the groups will determine which species should be favoured in the thinnings.

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[^0]:    ${ }^{1}$ Corresponding author: fuhrere@erti.hu; H-1023 BUDAPEST, Frankel Leó u. 42-44.

